Prof Day

THE ANALYST;

OR,

MATHEMATICAL MUSEUM.

CONTAINING

NEW ELUCIDATIONS, DISCOVERIES AND IMPROVEMENTS,

IN VARIOUS BRANCHES OF THE

MATHEMATICS,

WITH COLLECTIONS OF QUESTIONS

PROPOSED AND RESOLVED

BY INGENIOUS CORRESPONDENTS.

VOL. I. No. II.

UTILE DULCI.

PHILADELPHIA:

PUBLISHED BY WILLIAM P. FARRAND AND CO.

FRY AND KAMMERER, PRINTERS.

1808.

THE ANALYST,

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MATHEMATICAL MUSEUM.

VOLUME I. NUMBER II.

ARTICLE IV.

SOLUTIONS TO THE QUESTIONS PROPOSED IN ARTICLE III.

QUESTION I.

Solved by William Child, Pottsgrove, Pennsylvania.

SUPPOSE the sum of the required stakes to be 5*l.* to the triple of which adding 4, the sum 19 exceeds 9 by 10, which excess 10 should by the question be equal to twice the excess of 9 above 5, that is, to 8, therefore the error is 2. Again, suppose the stakes to be 4*l.* which trebled and augmented by 4 produce 16, which exceeds 9 by 7; but the excess of 9 above 4 being doubled gives 10, which by the question should be equal to 7, the preceding excess; in this case therefore the error is 3, and of a contrary kind to the former error; it is certain therefore that the answer is between the numbers 4 and 5, and since the errors are 2 and 3, we have only to divide the difference between 5*l.* and 4*l.* in proportion to these errors; thus, as the sum of 2 and 3, viz. 5 is to 3, so is 1*l.* to $\frac{3}{5}$*l.* or 12*s.*; therefore, the required sum of the stakes is exactly 4*l.* 12*s.*

QUESTION II.

Solved by John Gummere, near Burlington, New-Jersey.

Put $a=\frac{58}{2}=29$, and let $x+a=$ the number of men in the greater side of the rectangle, and since the difference of the sides of the rectangle is $58=2a$, the less side will be denoted by $x+a-2a=x-a$, and the whole number of men required will be $(x+a)\times(x-a)=x^2-a^2$, which according to the question must be an integral square number, because the number of men must be such that they can be formed into an exact square. Assume therefore x^2-

$a^2=(x-z)^2=x^2-2xz+z^2$, from which we obtain $2xz=z^2+a^2$, and $x=\frac{z^2+a^2}{2z}$. Now take $z=1$, then will $x=421$, and $x^2-a^2=176400=$ the number of men required: for since $a=29$ is a prime it is easy to perceive that 1, a, and a^2 are the only values we can possibly assume for z that will give x an integer; and of these three values of z the first 1, and third $(29)^2=841$ give the same value of x, the other value of z, viz. $a=29$, gives $x=a$ and $x^2-a^2=0$; therefore the number 176400 is the only answer of which the question is susceptible.

REMARK BY THE EDITOR.

We may prove that x^2-a^2 or $x^2-\overline{29}^2$ can only be a square when $x=421$ in the following manner: Put $x^2-a^2=y^2$, then $x^2-y^2=a^2$, that is $(x+y)\times(x-y)=a^2$, and $x+y=\frac{a^2}{x-y}$; it is certain therefore that $x-y$ must be some integral factor of a^2, and less than a, that the other factor $x+y$ may be greater than $x-y$; but since 29 is a prime there is no such factor of 29 $\times$29 excepting unity, therefore $x-y=1$, and $x+y=a^2$; whence $x=\frac{1+a^2}{2}=421$.

Another solution, by Ebenezer R. White, Danbury, Connecticut.

Let $a=$ the side of the required square, $a+z=$ one side of the required rectangle, and $a+z-58=$ its other side, then $(a+z)\times(a+z-58)=a^2$, that is, $a^2+2az+z^2-58a-58z=a^2$, from which $a=\frac{58z-z^2}{2z-58}$, and therefore $2z$ must be greater than 58, or z greater than 29. To obtain an integer for a take $z=30$, then $a=420$, consequently $a^2=176400=$ the number of men; also the sides of the rectangle $a+z$ and $a+z-58$ become 450 and 392 respectively, the product of which 420×392 exactly makes the square 176400, and their difference is 58.

Another solution, by John Willits, Mansfield, near Burlington, New-Jersey.

Let $x=$ the number of men in the less side of the rectangle, then $x+58=$the greater side; also put $y^2=$ the whole number of men, and by the question $x^2+58x=y^2$, whence $x^2+58x+841=y^2+841$, and $x=\sqrt{y^2+841}-29=\sqrt{y^2+b^2}-b$, putting $b=29$; we have therefore b^2+y^2 to be made a rational square, for which we have the following universal theorem, No. I. of the Analyst, $b:y::m^2-n^2:2mn$. Now as b is known, we have only to determine such values of m and n that $m^2-n^2=29$: but it is well known, that the difference of the squares of two numbers is equal to their

sum when their difference is unity; and the only two numbers whose difference is unity and sum 29 are 15 and 14; hence by taking $m=15$, $n=14$, the expressions m^2-n^2 and $2mn$ become 29 and 420, and $x=392=$ the less side, $x+58=$ the greater, and their product $=y^2=(420)^2=176400$ the number of men required.

QUESTION III.

Solved by Seth Smith, Burlington, New-Jersey.

Put $a=1000$ dollars, $r=1\frac{6}{36500}=$the amount of 1 dollar in a day at 6 per cent. $n=365$, and $x=$ the sum to be drawn daily. The amount of a in one day is ar, from which deducting x there remains $ar-x$ in the hands of the borrower at the end of the first day; again, the amount of $ar-x$ in one day being ar^2-rx, therefore ar^2-rx-x remains at the end of the second day; in like manner $ar^3-r^2x-rx-x$, and $ar^4-r^3x-r^2x-rx-x$ remain at the end of the third and fourth days, hence $ar^n-r^{n-1}x-r^{n-2}x-r^{n-3}x\ldots-x$ denotes the sum remaining at the end of the eleventh day; therefore by the question $ar^n-r^{n-1}x-r^{n-2}x-r^{n-3}x\ldots-x=o$, or $ar^n=x(r^{n-1}+r^{n-2}+r^{n-3}+\ldots+r+1)=x\left(\frac{r^n-1}{r-1}\right)$, whence $x=a\times\frac{\overline{r-1}\,.\,r^n}{r^n-1}=2.82296$ dollars as required.

QUESTION IV.

Solved by John Capp, the proposer, Harrisburg, Pennsylvania.

Put $a=100$ miles the distance between Philadelphia and Harrisburg, $c=4=$ the initial velocity, $x=$ any variable space travelled in the time z, and $v=$ the velocity of the traveller at the end of that time. Since the distance gone over is as the time when the velocity is uniform, we have $v:\dot{x}::1\text{ hour}:\dot{z}=\frac{\dot{x}}{v}$; but by the question as $\sqrt{a}:\sqrt{a-x}::c:v=\frac{c\sqrt{a-x}}{\sqrt{a}}$, whence by substitution $\dot{z}=\frac{\sqrt{a}\times\dot{x}}{c.\sqrt{a-x}}$, and taking the fluent we get $z=C-\frac{2\sqrt{a}\times\sqrt{a-x}}{c}$ in which C is an arbitrary constant quantity introduced to correct the fluent if necessary. To determine C we know that at the commencement of the fluent z and x are each $=o$, in which circumstance the general fluent $z=C-\frac{2\sqrt{a}\times\sqrt{a-x}}{c}$ becomes $o=C-\frac{2a}{c}$, therefore $C=\frac{2a}{c}$, and the correct fluent is $z=\frac{2a-2\sqrt{a}\times\sqrt{a-x}}{c}$,

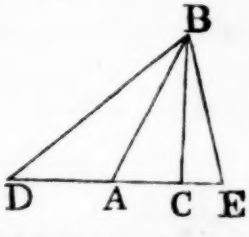
which by taking $x=56$ gives $z=16.8337521$ hours $=16$ hours 50 minutes $1\frac{1}{2}$ seconds the time required.

Another solution, by Daniel Smith, jun. Burlington, New-Jersey.

Let the area of the right angled triangle EPH represent the distance from Philadelphia to Harrisburg $=100$ miles, the similar right angled triangle SRH that from Reading to Harrisburg $=44$, and consequently the remainder EPRS that from Philadelphia to Reading $=56$; also let $EP=4=$ the velocity of the traveller at Philadelphia: and since by similar triangles SR is every where as the square root of SRH, it is evident from the data of the question that SR will represent the traveller's rate of motion corresponding to any distance from Harrisburg represented by the area SRH. Now it is demonstrated by the writers on mechanics that if EPRS denotes the space passed over by a moving body, and the perpendicular RS be taken to measure its velocity, the base PR will be the true measure of the corresponding time. Dividing 100 the measure of the area EPH by 2 which is half the perpendicular EP, we have $PH=50$ the number of hours in which the traveller would arrive at Harrisburg from Philadelphia: Again, by similar triangles, as the square root of the area EPH, viz. 10 is to the square root of the area SRH, viz. $\sqrt{44}$ or $2\sqrt{11}$, so is $PH=50$ to $RH=10\sqrt{11}=33.1662479$, which taken from $PH=50$ leaves PR $=16.8337521=16$ hours 50 minutes $1\frac{1}{2}$ seconds, the time required in travelling from Philadelphia to Reading.

QUESTIONS V. AND VI.

Solved by the Proposer Ebenezer R. White, Danbury, Connecticut.



Let the base AC of the right angled triangle ABC be produced to D, that DA may be equal to the hypothenuse $AB=h$, and join BD; then $BD^2=2h^2+2hz=2h(h+z)$, and $CD^2=(h+z)^2$, and as $BD^2 : CD^2 :: 1^2 : \sin.^2 CBD = \cos.^2 BDC = \cos.^2 \frac{1}{2} BAC = CD^2 \div BD^2 = (h+z)^2 \div 2h(h+z) = \frac{h+z}{2h}$, which is the first theorem.

Again, the same construction remaining, let ABE be an oblique angled triangle having the base $AE=b$, and side $BE=c$; then $AC=z=\frac{h^2+b^2-c^2}{2b}$, $h+z=\frac{2hb+h^2+b^2-c^2}{2b}=\frac{(h+b)^2-c^2}{2b}$; and by theorem first, $\cos.^2 \frac{1}{2} BAC=\frac{h+z}{2h}=\frac{(h+b)^2-c^2}{4bh}$, which is the second theorem.

The same questions solved otherwise by John Coope, Chester County, Pennsylvania.

Let the three sides of an oblique angled triangle be denoted by a, b, and c; s= the cosine of half the angle contained by a and b, the radius r being 1: then by Prop. VIII. Playfair's Plain Trigonometry, we have $4ab : (a+b+c)\times(a+b-c) :: r^2 : s^2$, from which $s^2=\frac{a^2+2ab+b^2-c^2}{4ab}=\frac{(a+b)^2-c^2}{4ab}$; but if the triangle be right angled then $a^2-c^2=b^2$, and $s^2=\frac{2ab+2b^2}{4ab}=\frac{a+b}{2a}$. Q. E. D.

QUESTION VII.

Solved by Samuel Cowgill, near Burlington, New-Jersey.

Let x= the greater segment, y= the less, then by the question $x^2y=b$, from which $y=\frac{b}{x^2}$; also per question $xy-y^2=c$, or (substituting $\frac{b}{x^2}$ for y,) $\frac{b}{x}-\frac{b^2}{x^4}=c$, which by reduction becomes $cx^4-bx^3+b^2=o$ from which x and every thing in the question may be found.

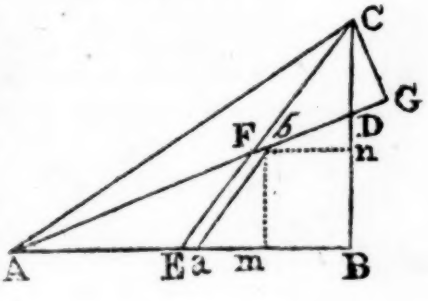
The same solved otherwise by the proposer, James M'Gennis, Harrisburg.

Put x= the greater segment, y= the less, then per question $xy.\ x=x^2y=b=47.014176$, $(x-y)y=xy-y^2=c=4.1712$, whence $x=\frac{y^2+c}{y}$, and squaring this last $x^2=\frac{y^4+2cy^2+c^2}{y^2}=\frac{b}{y}$, which reduced becomes $y^4=2cy^2-by+c^2=o$. This equation resolved gives $y=2.64$, $x=4.22$, and the base $=x+y=6.86$, also the perpendicular of the triangle $=\frac{2a}{x+y}=4.3$, (a being = area $=14.749$,) from which the sides are found to be 6.024813 and 5.045756 as required.

QUESTION VIII.

Solved by the proposer, John Gummere.

On AD produced let fall the perpendicular CG, make Ab equal to BC, and Aa a mean proportional between BC and CG, join ab, parallel to which draw CFE the line required.



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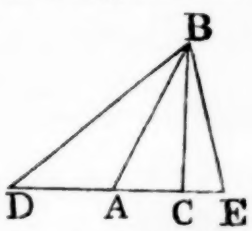

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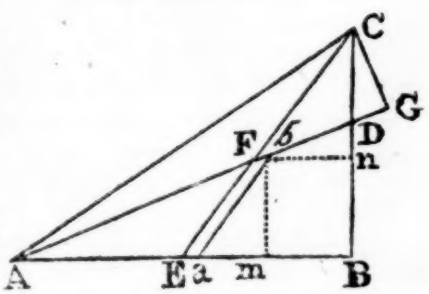

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to the square of Ab, that is, as CG to CD, that is, as the sine of CAF to the sine of CAE, which is compounded of the ratios of the sine of CAF to the sine of ACF, and of the sine of ACE to the sine of CAE; that is, of CF to AF, and of AE to CE, which compound ratio is that of the rectangle CF. AE to the rectangle AF. CE; therefore the square of AE is to the square of AF as the rectangle CF. AE to the rectangle AF. CE, whence AE is to AF as CF to CE, and therefore the rectangle AE. EC is equal to the rectangle AF. FC. Which was to be done.

CALCULATION.

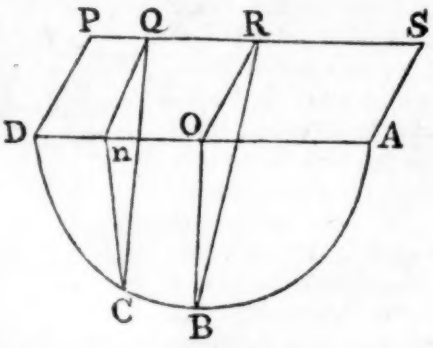
In the triangle AEF, we have the angle EAF and the ratio of the sides AE, AF, from which the angle AEF is found 129° 59′ $19\frac{1}{2}''$, and consequently BEC=50° 0′ $40\frac{1}{2}''$; hence in the right angled triangle BEC we have the angle BEC and side BC, whence EC is found 156.623, as required.

Another solution to the same, by John D. Craig, Baltimore.

By trigonometry AB (see the preceding figure) is found = 187.175=a, AD=194.221=c, BD=51.903=d, and BC is given =120=b. From F draw the perpendiculars Fm, Fn. Then since AE · EC=AF · FC, we have AE : AF : : FC : CE; but FC : CE : : Cn : CB, therefore AE · CB=AF · Cn. Now to find specific values for these quantities put AF=x, and by sim. tri. $c : a : : x : \frac{ax}{c}$= Am; hence $a-\frac{ax}{c}$=Bm=Fn; and as $c : d : : x : \frac{dx}{c}$=Fm=Bn, hence $b-\frac{dx}{c}$=Cn; moreover, as Cn : Fn : : CB : BE=$\frac{ab \cdot \overline{c-x}}{bc-dx}$, whence AE=$\frac{ax \cdot \overline{b-d}}{bc-dx}$, and our equation becomes $\frac{ax \cdot \overline{b-d}}{bc-dx} \times b = x \times \overline{b-\frac{dx}{c}}$; from which by completing the square, &c. we find x to be =116.982, and CE=156.6.

PRIZE QUESTION.

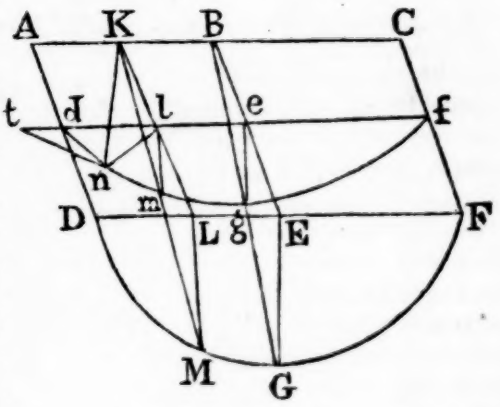
Solution to the Prize Question, by John D. Craig, Baltimore.



Let the semicircle ABCD represent half the circular base on which the roof stands. Let PQRS be the ridge, equal to, and directly over the diameter DOA; from the centre O draw the lines OR and OB, each perpendicular to DOA; from any point n in DO draw nQ, nC parallel to OR, OB, and join RB, QC, and PD. Put OD, OR, or OB=a (50,) and Dn, supposed to be variable, =x; then, by the nature of the circle, Cn=$\sqrt{2ax-x^2}$; and Cn×Qn=$a\sqrt{2ax-x^2}$, the area of the whole triangle by which the solid capacity may be conceived to be generated, while moving from D to A. Therefore $\dot{a}x\sqrt{2ax-x^2}$ is the fluxion of the solid content, whose fluent, when n coincides with A is =a× semicircle DAB. Hence it appears that the capacity of the roof is half that of its circumscribing cylinder. The same conclusion might be had by supposing the generating plane to move parallel to the base; it being easily proved that such a plane would be an ellipsis, whose transverse is DA, and semiconjugate equal to the plane's distance from the ridge PS.

Thus far Mr. Craig. Mr. Gummere gave a geometrical investigation of the solidity by the method of ultimate ratios. Messrs. Capp and M'Gennis also determined the solid capacity of the roof by Cavaleni's method of indivisibles: all these agree with Mr. Craig in demonstrating that the solid capacity of the roof is precisely half the circumscribing cylinder. But notwithstanding the attempts of several contributors, the Editor has received no true investigation of the surface of the roof, nor even a hint of the very curious and interesting principle on which a true solution must necessarily be founded; although the hope of seeing this important principle developed and exemplified was the only motive which induced him to propose the problem as the subject of the prize.

Solution to the Prize Question by the Editor, Robert Adrain.



Let ACFD be a vertical plane section of the roof through the ridge AC and diameter DF of the tower's upper base, of which DGF is the semicircumference; also let defgd be a plane section of the figure parallel to DEFGD, and BEG, KLM two vertical plane sections of it at right angles to AC: and since by the question BG, KM are straight lines, therefore BEG, KLM are evidently right angled plane triangles; and if eg, lm be the common sections of those triangles with the plane dfg, the triangles Beg, Klm will also be right angled at e and l, and be similar to BEG, KLM. Suppose BEG to pass through E the centre of DGF; since therefore by similar triangles EG is to LM, as eg to lm, it is manifest that dgf is a semiellipsis of which the greater axis df is equal to DF, and the less semiaxis eg equal to Be, for by the question BE is equal to EG.

Let the straight line mt touch the ellipsis in m, and meet ed produced in t, from l draw ln perpendicular to the tangent mt, and join Kn.

Suppose now that the solid content of the semiroof is generated by the motion of the right angled triangle KLM continuing parallel to BEG while it proceeds from CF to AD, and its surface will evidently be generated by the motion of the hypothenuse KM. Because the generating triangle has a given altitude KL, the fluxion of the solid is equal to the fluxion of the base FLM multiplied by half this given height; and therefore the solid content of the part on the base FGML is half the prismatic solid on the same base, and of the same altitude BE; of course the solid content of the whole roof is half the circumscribing cylinder, or is equal to a paraboloid of the same base and altitude: the solidity required is therefore .78539, &c. $\times 2\overline{DE}^2 = .78539$, &c. $\times 25000 = 196349.54$ cubic feet.

To obtain the fluxion of the surface generated by KL, it is evident that we must multiply each particle p of KL by the velocity of that particle estimated at right angles to its length, in other words, we must multiply the fluxion of Km (considered as variable) by the velocity of that fluxion taken at right angles to its length, and the correct fluent of this fluxion to any length Km will

be the fluxion of the surface generated by Km, and the whole fluent will be the true fluxion of the surface generated by the whole length KM: to this we beg leave to add, as by far the most valuable information contained in the present solution, that this principle enables us to determine the measure of all curve superficies, whatever be their figure, or the nature of the generating line KM.

Put BE=EG=1, EL=x, LM=$y=\sqrt{1-x^2}$, Kl=u, and $\dot{x}=c$= the uniform velocity of the point L along ED; then lm=uy, Km=$u\sqrt{1+y^2}$, et=$\frac{1}{x}$, lt=$\frac{1}{x}-x=\frac{1-x^2}{x}=\frac{y^2}{x}$, tm=$\sqrt{\text{tl}^2+\text{lm}^2}$ =$\frac{y}{x}\sqrt{y^2+x^2u^2}$, ln=$\frac{\text{tl}\times\text{lm}}{\text{tm}}=\frac{uy^2}{\sqrt{y^2+x^2u^2}}$, Kn=$\sqrt{\text{Kl}^2+\text{ln}^2}=u\frac{(y^2+y^4+x^2u^2)^{\frac{1}{2}}}{(y^2+x^2u^2)^{\frac{1}{2}}}$; and therefore in the plane triangle Kmn right-angled at n, we have the sine of the angle Kmn=$\frac{\text{Kn}}{\text{Km}}=\frac{(y^2+y^4+x^2u^2)^{\frac{1}{2}}}{(1+y^2)^{\frac{1}{2}}\times(y^2+x^2u^2)^{\frac{1}{2}}}$. Again, as tl : tm :: the vel. of KL, viz. c:$c\frac{(y^2+x^2u^2)^{\frac{1}{2}}}{y}$= the velocity of the particle p at m in the direction of the tangent mt; and this velocity multiplied by the sine of Kmn gives $c\frac{(y^2+y^4+x^2u^2)^{\frac{1}{2}}}{y(1+y^2)^{\frac{1}{2}}}$ for the velocity of p at right angles to its length: now let this last velocity be multiplied by the length of the particle p, and we shall have $cp\times\frac{(y^2+y^4+x^2u^2)^{\frac{1}{2}}}{y(1+y^2)^{\frac{1}{2}}}$= the element of the fluxion of the surface produced by the particle p. Let p now be taken = the fluxion of Km=$\dot{u}(1+y^2)^{\frac{1}{2}}$, while x and y have any fixed values m and n, and the preceding element becomes $\frac{c\dot{u}}{y}(y^2+y^4+x^2u^2)^{\frac{1}{2}}=\frac{c\dot{u}}{n}(n^2+n^4+m^2u^2)^{\frac{1}{2}}=\frac{mc}{n}\times\dot{u}(a^2+u^2)^{\frac{1}{2}}$, (putting $a^2=\frac{n^2+n^4}{m^2}$,) of which the correct fluent is $\frac{mc}{n}\times\left\{\frac{1}{2}u(a^2+u^2)^{\frac{1}{2}}+\frac{1}{2}a^2\times\text{hyp. log.}\frac{u+(a^2+u^2)^{\frac{1}{2}}}{a}\right\}$: and when u=KL=1, this fluent becomes $\frac{mc}{n}\left\{\frac{1}{2}(a^2+1)^{\frac{1}{2}}+\frac{a^2}{2}\text{ h. log.}\frac{1+(a^2+1)^{\frac{1}{2}}}{a}\right\}$; which fluent by restoring the values of m, n and c, gives $\dot{x}\times\frac{1}{2}\left\{(1+y^4)^{\frac{1}{2}}+\frac{y^2+y^4}{x}\times\text{ h. log.}\frac{x+(1+y^4)^{\frac{1}{2}}}{(y^2+y^4)^{\frac{1}{2}}}\right\}$ for the true fluxion of the surface generated by KM, while x acquires the fluxion $\dot{x}$; and the

fluent of this fluxion being taken from $x=0$ to $x=1$, and quadrupled will give the surface required.

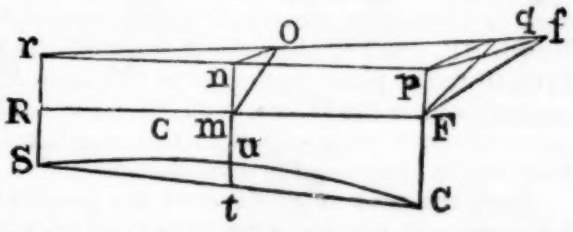
It only remains to determine the fluent of the fluxion just found; but as this fluxion is not susceptible of a rigorous integration in algebraic terms, nor even by circular arcs and logarithms, we must have recourse to some of the known methods of approximation. As the fluxional formula is somewhat complex it would be very laborious to compute the fluent in a direct manner from infinite series; we shall therefore consider the required fluent as the area of a plane curvilineal figure, and calculate that area by the differential method. It is evident that the fluent sought is the area of a curve on the abscissa x, having the corresponding rectangular ordinate $=\frac{1}{2y}\times\left\{(1+y^4)^{\frac{1}{2}}+\frac{y^2+y^4}{x}\times \text{h. log.}\frac{x+(1+y^4)^{\frac{1}{2}}}{(y^2+y^4)^{\frac{1}{2}}}\right\}$; but on examining the nature of this curve we find that as x approaches indefinitely to the value ED, the corresponding ordinate increases indefinitely; so that the curve is assymptotic, and its area cannot be found by the common differential method without certain auxiliary and embarrassing researches. To avoid this difficulty therefore let Z= the circular area GM to the sine x, also put s= the surface BGMK, and $\dot{Y}=\frac{1}{2}\times\left\{(1+y^4)^{\frac{1}{2}}+\frac{y^2+y^4}{x}\times \text{h. log.}\frac{x+(1+y^4)^{\frac{1}{2}}}{(y^2+y^4)^{\frac{1}{2}}}\right\}$: and since $\dot{Z}=\frac{\dot{x}}{y}$, we shall evidently have $\dot{S}=Y\dot{Z}$; from which equation it is plain that the value of S is the area of a plane curve of which the abscissa is every where equal to Z, and the corresponding rectangular ordinate equal to Y. Now on searching the ordinates belonging to the several points of this new curve, we find that the greatest belongs to G the beginning of Z, and is equal to BG$=\sqrt{2}$, and the least (when Z=GD) is the half of AD $=\frac{1}{2}$; of course we may determine the area of this curve to any required degree of accuracy by equidistant ordinates: we shall make use of seven ordinates which will give the number of square feet required to the nearest unit.

Assume Z$=\left|\begin{matrix}00^\circ\\15^\circ\\30^\circ\\45^\circ\\60^\circ\\75^\circ\\90^\circ\end{matrix}\right|$, and by calculation, Y$=\left|\begin{matrix}1.4142136\\1.3512227\\1.1810392\\0.9543772\frac{1}{2}\\0.7358176\\0.5749816\\0.5000000\end{matrix}\right|$

Put now A= the sum of the first and last ordinates $=1.9142136$, B= the sum of the second and sixth $=1.9262043$, C= the sum of the third and fifth $=1.9168568$, and D= twice the fourth $=1.9087545$; and by the rule for seven ordinates (see Prop. II. Sec. 11. Part IV. Hutton's Mensuration) we have the whole value of S

$= \frac{41A+216B+27C+136D}{840} \times GMD = .95939625 \times GMD$: but the area of the quadrant DEG $= \frac{1}{2} \times DMG$, and therefore the area of the whole circular base of the roof is to the whole superficies of the roof as $\frac{1}{2}$ to .95939625, or as unity to 1.9187825; this last therefore multiplied by 3.1415926×2500 gives the true value of the required surface $= 15070$ square feet.

Another solution to the same by the Editor, Robert Adrain.



Let RF, rf be two straight lines representing rafters, and intercepting between them an indefinitely narrow element RFfr of the surface of the roof, with Rr and Ff the corresponding elementary particles of the straight ridge and circular eaves: now it is evident that we can solve the problem if we can truly determine the measure of the elementary surface RFfr corresponding to every element Rr of the ridge; for we shall then have only to find the fluent of a certain fluxion, or which amounts to the same thing, we shall have to find the area of a certain plane curvilineal figure, which can always be effected by some of the known methods for the exact or approximate quadratures of curves.

Since Rr is parallel to the upper base in which F is situated, let Fp be drawn in the plane of that base equal and parallel to Rr, and draw the straight lines rp, pf, from which construction it is plain that RFpr is a rectangle, and that Fp, pf are the elementary variations of the sine and cosine of the circular eaves, reckoning this circular arc from the point where the rafter is a maximum; so that Fpf is a plane triangle, right-angled at p.

It is evident that Rr, being at right angles to both the straight lines RF, rf, is the true breadth of the elementary superficies RF fr at its extremity R: let us determine its breadth at the other extremity F. For this purpose draw in the plane of the triangle prf the straight line pq at right angles to rp, and join Fq which is the required breadth at F: for since rp, rf are each at right angles to Rr, the plane of the triangle rpf is at right angles to the plane of the rectangle Rp, therefore pq, and consequently the plane of the triangle pFq are at right angles to Rp; it is manifest therefore that RF is perpendicular to the plane pFq, and of course Fq is at right angles to RF, and is the true breadth of the elementary surface RFfr at F. It is obvious also that the triangle Ffq is an indefinitely small part of the whole element RFfr; every one there-

fore who is acquainted with the higher geometry will perceive that we have only to determine the content of the figure RFqr. We see then that the real difficulty and originality of the problem consist in determining the true measure of the figure RFqr.

As the breadth Eq of the superficies RFqr, although coincident with that superficies, is indefinitely little, it may be taken for a straight line; the indefinitely narrow superficies RFqr is therefore bounded by four straight lines, two of which Rr and Fq are at right angles to the side RF; and some might perhaps think that the figure is simply a plane trapezoid, and that therefore the area is accurately measured by the rectangle contained by the length RF, and half the sum of the extreme breadths Rr, Fq, or is equal to the plane right-angled trapezoid RPGS, of which the perpendiculars RS FG are respectively equal to Rr and Fq; but a more accurate examination of this singular figure will teach us that such a conjecture is materially defective.

From any point m of RF draw mn parallel to Rr, and no parallel to pq, join mo, and let mt produced meet the straight line SG in t; it is obvious that mo is the true breadth of the element RFqr at m. Now it is certain that the breadth mo is less than mt the breadth of the trapezoid RFGS in a certain assignable ratio; so that if we compute the two areas Rmor, mFqo, on the supposition that they are plane trapezoids, their sum will be less than the former measure of the whole RFqr computed on the same supposition, in a certain finite assignable ratio: this mode of calculation therefore contradicts itself, and consequently cannot possibly be true. In a word, it is plain that if we would measure RFfr with accuracy we must examine its breadth mo in every part of its length, and this examination conducts us to a very curious result; for by making in the plane of the trapezoid RFGS the perpendicular mu every where equal to the corresponding breadth mo, we discover that the extremities of all these perpendiculars, instead of falling on the straight line SG, lie in a certain curve line SuG convex to RF. This curve is in fact a common hyperbola having its vertex in S, its semitransverse axis equal to RS or Rr, and its semiconjugate axis equal to a certain finite straight line RC, which we shall presently assign. Now as the corresponding breadths mo, mu of the indefinitely narrow figures RFqr and RFGuS are every where equal, it is evident that the area Rmor is equal to the area RmuS, and the whole element of the required superficies RFqr equal to the whole hyperbolic area RFGuS. These equalities however are only to be understood of the two figures as the limit of their ratio when the breadths mo, mu are diminished indefinitely, for when mo, mu have finite values, the corresponding surfaces Rmor and RmuS are by no means equal, as will readily appear to the intelligent geometer.

Let us prove now that SuG is a hyperbola. Make RC the tangent of the angle pqF to the radius RF; and because no is to pq as Rm to RF, and pq to pF as RF to RC; therefore no is to Fp or RS as Rm to RC; and therefore the square of no, that is, the difference of the squares of mu, RS is to the square of RS, as the square of Rm to the square of RC, which is the known property of a hyperbola having the vertex in S the centre in R, and RS, RC the semitransverse and semiconjugate axes. To this let us add for the sake of beginners, that the hyperbolic area RFGuS is by no means ultimately in the ratio of equality with the trapezoid RFGtS: for it is well known from the nature of the hyperbola, that while the base RF and semiconjugate axis RC remain the same, the ratio of the hyperbolic area RFGuS to the trapezoid RFGtS will also remain the same, whatever be the magnitude of the semitransverse RS: so that if we determine the ratio of these two figures when the semitransverse axis is taken equal to the semiconjugate axis RC, that is when the hyperbola is equilateral, and the figures both finite; the same precisely will be their ratio when the semiconjugate axis RS, as well as the figures RFGuS, RFGtS themselves are all diminished ad infinitum. We have therefore truly determined the measure of the element of the required surface; for we have shown that it is equal to a certain hyperbolic area, and the quadrature of the hyperbola is a subject which has long been familiarly known to geometers.

Having thus discovered the value of the fluxion of the surface of the roof, we find no difficulty in perceiving that the surface itself is equal to twice the area of a certain plane curvilineal figure on an axis equal to the length of the ridge, and having on every particle of this axis Rr, an increment or fluxion equal to the hyperbolic area RFGuS; or, which is the same in fact, having for its perpendicular ordinate at R the length of a rectangle equal to the area RFGuS on the base Rr.

Let us now examine the ordinates of this plane figure of which the area gives the surface required. In the first place it is evident that when the rafter RF is a maximum, that is, when it proceeds from the middle of the ridge, the angle Ffp becomes right, therefore the semiconjugate axis RC of the hyperbola RuS becomes indefinitely great, and therefore the curve line RuS degenerates into a straight line, so that in this case the ordinate of the curve is equal to the length of the rafter. As the point R recedes from the middle of the ridge, the area RFGuS and ordinate of the curve at first decrease, (the particle Rr being given or constant,) and passing through a minimum, they afterwards increase continually: and when the rafter is a minimum, that is, at the extremity of the ridge, the angle Ffp vanishes, and also the semiconjugate axis RC, the curve RuG therefore

in this case coincides with its axis RS produced, and the area RFGuS becomes infinite, which teaches us that the ordinate of the curve at the extremity of the ridge is also infinite. This curve therefore when described on both sides of its axis has four similar and equal branches extending indefinitely along four asymptotes at right angles to the axis, and passing through its extremities.

But as the measure of these asymptotic areas could not be obtained in a direct manner without considerable difficulty, on account of the manner in which the ordinates must be found, we may with advantage in this as in many other cases have recourse to some other curve, such that the element of the axis in this new curve may be indefinitely greater than the corresponding element Rr in the preceding curve, in order if possible to avoid the determination of asymptotic spaces. Let us therefore make the semicircumference of the upper base of the tower the axis of our curve; and raising at every equal particle Ff of the base or axis, an ordinate, such that the rectangle contained by it and the element Ff may be equal to the elementary superficies RFfr, or RFGuS, we obtain a curve of which the greatest ordinate is at its middle, and is equal to the greatest rafter, and its least ordinate at either extremity equal to half the least rafter; and the double area of this curve being computed by any of the known methods of approximation will give the surface required.

It is unnecessary to enter into a detail of the computation of this latter curve; in fact this has already been performed in the preceding solution of the problem, for there is the most exact coincidence between the result of this geometrical investigation and the former by algebra: and on computing algebraically the measure of the hyperbolic area RFGuS, which is the fluxion of the required surfaces, we shall find it to agree precisely with the formula in the foregoing solution which is there derived from the genuine principles of fluxions. The algebraic expression of the area RFGuS is easily found in the following manner. Put x = the distance from the middle of the ridge to the point r, or the sine of the circular eaves reckoning from the point where the rafter is a maximum, y= the cosine of the same arc, the radius being unity; and we have Rf or RF$=(1+y^2)^{\frac{1}{2}}$=r, Rr=$\dot{x}$=a, pf$=\frac{ax}{y}$; and since the perpendicular from r on pf produced is unity, we have pq$=\frac{ax}{y}\div$r$=\frac{ax}{ry}$; also as pq : pF : : RF : RC$=\frac{r^2y}{x}$ =b. Having now a, and b, the semitransverse and semiconjugate axes of the hyperbola SuG, we can easily find its area corresponding to any proposed abscissa RF; for putting Rm=v, and mo=

mu=u, we have $u^2-a^2 : a^2 :: v^2 : b^2$, whence $u=\frac{a}{b}.(v^2+b^2)^{\frac{1}{2}}$, and therefore the fluxion of the area Rmor, or RmuS is $\dot{uv}=\frac{a\dot{v}}{b}.(v^2+b^2)^{\frac{1}{2}}$, of which the correct fluent $=\frac{a}{2b}.\left\{v(v^2+b^2)^{\frac{1}{2}}+b^2.\text{ h. log. }\frac{v+(v^2+b^2)^{\frac{1}{2}}}{b}\right\}$ = the true value of the area Rmor, or RmuS; and when $v=r$, the area RFGuS$=\frac{a}{2b}.\left\{r(r^2+b^2)^{\frac{1}{2}}+b^2.\text{ h. log. }\frac{r+(r^2+b^2)^{\frac{1}{2}}}{b}\right\}$. Now we easily obtain $\frac{a}{2b}=\frac{ax}{r^2y}$, $(r^2+b^2)^{\frac{1}{2}}=\frac{r}{x}(1+y^4)^{\frac{1}{2}}$, $b^2=\frac{r^4y^2}{x^2}$, and $\frac{r+(r^2+b^2)^{\frac{1}{2}}}{b}=\frac{x+(1+y^4)^{\frac{1}{2}}}{(y^2+y^4)^{\frac{1}{2}}}$, therefore by substitution we have the true value of the area RFGuS, or RFqr $=\frac{\dot{x}}{y}.\frac{1}{2}\left\{(1+y^4)^{\frac{1}{2}}+\frac{y^2+y^4}{x}\times\text{h. l.}\frac{x+(1+y^4)^{\frac{1}{2}}}{(y^2+y^4)^{\frac{1}{2}}}\right\}$, agreeing precisely with the fluxion of the surface exhibited in the preceding solution.

ACKNOWLEDGMENTS, &c.

A list of the names and places of residence of the ingenious gentlemen who have favoured the Editor with solutions to the questions proposed in No. I. The figures annexed to the names refer to the questions answered by each as numbered in the work.

John Capp, Harrisburg, Pennsylvania,	1 2 3 4 5 6 7 8 9
William Charington, Reading, Pennsylvania,	1 3 4
William Child, Pottsgrove, Pennsylvania,	1
John Coope, Chester County, Pennsylvania,	1 2 3 5 6
Samuel Cowgill, near Burlington, New-Jersey,	1 2 3 4 5 6 7 8
John D. Craig, Baltimore,	1 2 3 4 5 6 7 8 9
James M'Ginnis, Harrisburg, Pennsylvania,	1 2 3 4 5 6 7 8 9
John Gummere, near Burlington, New-Jersey,	1 2 3 4 5 6 7 8 9
Charles Richards, Reading, Pennsylvania,	1 2 3 5 6 7
Daniel Smith, junr. Burlington, New-Jersey,	1 2 3 4 5 6 8
Seth Smith, Burlington, New-Jersey,	1 2 3 4 5 6 7 8
Ebenezer R. White, Danbury, Connecticut,	1 2 5 6
John Willits, Mansfield, near Burlington, New-Jersey,	1 2 4 5 6

As no complete solution of the prize question has been received, of course there is no prize medal awarded for this time.

The Editor begs leave to suggest to contributors the propriety of finishing their solutions in the neatest manner, so as to

render them fit for the public eye without the necessity of correction or amendment; such as observe this caution may depend on seeing their solutions published as often as room will permit. He hopes also that they will not be sparing in their communications of original problems, particularly such inquiries as may contribute to the improvement of science, or be acceptable to men of judgment and taste, by their ingenuity or elegance.

ARTICLE V.

SOLUTION OF CERTAIN EXPONENTIAL EQUATIONS,

BY ROBERT ADRAIN.

The method here given for the resolution of certain exponential equations consists in taking the logarithms of logarithms, as often as may be necessary to reduce to addition and subtraction those tedious multiplications and divisions which must be performed when we make use of logarithms only once: having by this means reduced the proposed equation to its simplest form we obtain the value of the unknown quantity with facility by trial and error, or any of the common methods of approximation. In giving a few examples of this method I shall denote the logarithm of x by lx, and the second logarithm of x, or the logarithm of lx by llx, &c.

PROBLEM I.

Given $x^x = a = 100$, to investigate the value of x.

SOLUTION.

Taking the logarithm of each side of the proposed equation we have $x \times lx = la$; and again, taking the logarithm of each side of the equation $x \times lx = la$, we have $lx + llx = lla$: put now $lx = y$, then $llx = ly$, put also $lla = b = l2 = .3010300$, and we have the equation $lx + llx = lla$ transformed into $y + ly = b$, or $y + ly - b = o$. It is evident that the equation $y + ly = b$, or $y + ly - b = o$, exhibits only the operations of addition and subtraction, with a table of logarithms, and of course the value of y is easily found as follows. By a few trials with a table of logarithms we discover that y is between the numbers .5 and .6;

Let us therefore suppose	$y =$ .5500000
And by the table of logarithms we have	$ly = -.2596373$
We have also	$b =$ .3010300
Whence by addition and subtraction	$y + ly - b = -$ 106673

Let us now assume	$y=$	.5600000
And by the table of logarithms	$ly=$	—.2518120
Whence by addition and subtraction	$y+ly-b=$	+ 71580
Say now, as the diff. of errors 178253 to diff. of suppositions 100000, so is the error 71580 to the proportional correction 40156, which taken from the second supposition gives $y=.5559844$.		
Suppose now	$y=$	.5559800
By the table of logarithms	$ly=$	—.2549408
Whence by addition and subtraction	$y+ly-b=$	+ 92
Suppose again	$y=$	.5559700
By the table of logarithms	$ly=$	—.2549486
Whence by addition and subtraction	$y+ly-b=$	— 86
Say now as the diff. of errors 178 to diff. sup. 100, so is the error 86 to the correction		48
Which added to the second supposition gives the true value of	$y=$	.5559748
And therefore the required value of	$x=$	3.5972845

Another solution of the equation $y+ly=b$.

Having found by trials a number n nearly equal to y, assume $n+v=y$, and by substitution the equation $y+ly=b$ becomes $n+v+l(n+v)=b$; but when v is very small in comparison to n, we know from the theory of logarithms that $l(n+v)=ln+\frac{mv}{n}$ very nearly, in which $m=.43429448$, &c. therefore the equation $n+v+l(n+v)=b$ becomes $n+v+ln+\frac{mv}{n}=b$, from which we obtain $v=n\times\left\{\frac{b-n-ln}{m+n}\right\}$. Assume now $n=.55$, and we have $v=\frac{.55\times.01067}{.984}=.00596$, therefore $n+v=.55596$; which last being put for a new value of n, we find the corresponding value of $v=\frac{.5559\times.0000265}{9903}=.0000148$, from which $n+v=y=.5559748$, the same as before.

REMARK.

The preceding problem is only a particular case of the general equation $(mx)^{nx}=a$, in which m and n are any given numbers. This general equation may be treated in the same manner. Taking the logarithm of each side of the equation, we have $nx\times lmx=la$; and taking the logarithm of this last we have $ln+lx+llmx=lla$; by transposition $lx+llmx=lla-ln$, by addition, $lm+lx+llmx=$

$lm-ln+lla$, that is, $lmx+llmx=lm-ln+lla$: now put $lmx=y$, then $llmx=ly$, put also $b=lm-ln+lla$, and we have $y+ly=b$ as in the preceding solution.

We may however arrive at this conclusion in a very different manner; for $(mx)^{nx}=a$ becomes $(mx)^{x}=a^{\frac{1}{n}}$ by extracting the n^{th} root; and by raising each side of this last to the m^{th} power we obtain $(mx)^{mx}=a^{\frac{m}{n}}$, which formula is now precisely the same as in problem I. So that putting $lmx=y$, and $lla^{\frac{m}{n}}=b$ we shall have $y+ly=b$.

The judicious reader will perceive the advantage of our method of resolving the equation $(mx)^{nx}=a$, when m and n are decimals or surds such as $\surd 2$, $\surd 3$, &c.

PROBLEM II.

Given $x^{x^{x}}=a=123456789$, to find the value of x.

SOLUTION.

Taking the logarithm of the equation proposed, we have $x^{x}\times lx=la$, and taking also the logarithm of this last, we have $lx^{x}+llx=lla$, that is, in effect, $x\times lx+llx=lla$, whence $x\times lx=lla-llx$; and taking the logarithm of this equation, we have $lx+llx=l(lla-llx)$: let us now put $lx=y$, then $llx=ly$; put also $lla=b=.9080298$, (for $la=8.0915150$) and by substitution we have $y+ly=l(b-ly)$, or $y+ly-l(b-ly)=o$.

By a few trials we discover that the value of y is between .4 and .5.

Let us suppose therefore	$y=$	.4500000
therefore by the table of logarithms	$ly=$	—.3467875
consequently	$y+ly=$	.1032125
and since	$b=$	.9080298
therefore also	$b-ly=$	1.2548173
and by the table of logarithms	$l(b-ly)=$	.0985805
therefore by subtraction	$y+ly-l(b-ly)=$	+ 46320

From this error we conclude that the assigned value of y is too great;

Let us therefore suppose	$y=$	.4400000
and proceeding as above we have	$y+ly-l(b-ly)=$	— 184926

Now from the errors +46320 and —184926 we discover by a stating in the rule of three that the correction answering to the first error is 20030

which by subtraction gives the value of $y=$ .4479970

Assume now $y=$ .4479900
and proceeding as above we find $y+ly-l(b-ly)=$ + 54

And trying $y=$ .4479800
we find $y+ly-l(b-ly)=$ — 176
Say now as the diff. errors 230 is to the diff. sup.
100, so is the error 54 to the correction 23
which taken from the value of $y=$ .4479900
leaves the correct value of $y=$ .4479877
whence by the table of logarithms $x=$ 2.805354

PROBLEM III.

Given $(mx)^{(nx)^{rx}}=a$, to investigate the value of x.

SOLUTION.

Taking the logarithm of each side of the proposed equation we have $(nx)^{rx}\times lmx=la$, and again proceeding to logarithms we have $l(nx)^{rx}+llmx=lla$; but we know that $l(nx)^{rx}=rx\times lnx$, and that $llmx=l(lm+lx)$, whence by substitution, we have $rx\times lnx+l(lm+lx)=lla$; and therefore $rx\times lnx=lla-l(lm+lx)$: let us now take the logarithm of this last and we have $lrx+llnx=l\left\{lla-l(lm+lx)\right\}$, which we may write thus $lr+lx+l(ln+lx)=l\left\{lla-l(lm+lx)\right\}$, or, which is the same in fact, $lr-lm+lm+lx+l(ln-lm+lm+lx)=l\left\{lla-l(lm+lx)\right\}$. Now put $lla=b$, $lr-lm=c$, $ln-lm=d$, $lm+lx=y$, then $l(lm+lx)=ly$, and by substitution we have $c+y+l(d+y)=l(b-ly)$; from which equation we may easily determine the value of y by addition and subtraction as in the preceding problems when m n r and a are any given numbers.

PROBLEM IV.

Given $x^{x^{x^{x}}}=10$, to determine the value of x.

SOLUTION.

Taking the logarithm of the given equation we have $x^{x^{x}}\times lx=1$, of which the logarithm is $x^{x}lx+llx=o$; put $lx=y$, and $x^{x}lx+llx=o$ becomes $x^{x}y+ly=o$, consequently $x^{x}y=-ly$, of which equation the logarithm is $lx^{x}+ly=$ log. of $-ly$, or $x\times lx+ly=$ log. of $-ly$, or finally $xy+ly=$ log. of $-ly$; this by transposition

becomes $xy=-ly+$ log. of $-ly$, and taking the logarithm of each side, we have $lx+ly=l(-ly+$ log. of $-ly)$, that is $y+ly=l(-ly+$ log. of $-ly)$, therefore $y+ly-l(-ly+$ log. of $-ly)=o$.

Having thus reduced our equation to its simplest form, we may determine the value of y by addition and subtraction with very little trouble as follows.

By a few trials we discover the value of y to be between .2 and .3, but nearer to the former,

Let us therefore assume	$y=$	.2400000
and by the table of logarithms	$ly=$	—.6197888
therefore	$y+ly=$	—.3797888
also	$-ly=$	.6197888
and by the table of logarithms	log. of $-ly=$	—.2077562
therefore	$-ly+$ log. of $-ly=$	.4120326
therefore by the table of logarithms	$l(-ly+$ log. of $-ly)=$	—.3850684
and therefore	$y+ly-l(-ly+$ log. of $-ly)=$	+ 52796

consequently our assumed value of y is too great.

Let us therefore suppose	$y=$	.2390000
and proceeding as above we find	$y+ly-l(-ly+$ log. of $-ly)=$	— 7701

Now as the diff. of errors 60497 is to diff. of suppositions 1000, so is the error 7701 to the correction 1273, which added to .2390000 gives the true value of $y=$ .2391273

from which by the table of logarithms we find $x=$ 1.7343125

ARTICLE VI.

NEW QUESTIONS,

TO BE ANSWERED IN THE NEXT NUMBER.

I. QUESTION 10.

By John Capp, Harrisburg.

In mixing together a quantity of alkohol and clear water of the temperature of sixty degrees, their respective weights being in the ratio of 3.3 to 4. I found the specific gravity of the mixture to be .9388; required the number of gallons of each ingredient in their unmixed state that would be necessary to compose a barrel of this mixture.

II. QUESTION 11.

By Jacob Conklin, jun. Bergen County, New-Jersey.

It is required to lay out 100 acres of land in a trapezium of which the sides shall be as 1, 2, 3, 4 respectively, and confined to a diagonal of 100 four-pole chains.

III. QUESTION 12.

By John Gummere, near Burlington, New-Jersey.

To find a point in the circumference of a given circle, from which two straight lines being drawn to two given points may have the sum of their squares a maximum, or minimum.

IV. QUESTION 13.

By Robert Patterson, Philadelphia.

In examining some ancient ruins, a horizontal sundial was found, but so much mutilated and defaced that the only datum which could be conveniently obtained was, that the angle contained between the meridian and the three-o'clock hour line exactly equaled that contained between the latter and the five-o'clock hour line. The latitude of the place for which this dial was made is required.

V. QUESTION 14.

By the same.

Suppose two straight thin strips of metal, the one steel and the other brass, to be soldered or otherwise fastened together in close contact; the length of each strip 6 inches, and its thickness .01 part of an inch: into what arc or portion of a circle will this compound strip be bent by an increase of temperature of 10 degrees, (Fahr. scale); supposing, according to Smeaton's experiments, that a piece of steel one foot in length expands .0138 parts of an inch, and a like piece of brass .0232 parts of an inch, by an increase of temperature of 180 degrees?

VI. QUESTION 15.

By John Eberle, Philadelphia.

To divide a globe into three equal parts by two parallel planes.

VII. QUESTION 16.

By John D. Craig, Baltimore.

On what day in the present year does the sun's apparent diameter, as seen from the earth, increase the fastest?

VIII. QUESTION 17.

By John Capp, Harrisburg.

To find three numbers such, that, when each is multiplied by the sum of the squares of the other two, the three products may be equal respectively to three given numbers a, b, and c; and to give an example when $a=121104$, $b=121632$, and $c=122400$.

IX. QUESTION 18.

By Enoch Lewis, Westown, Pennsylvania.

Required to apply geometrically to one of the parallel sides of a given trapezoid a figure equiangular to the given one, and having a given ratio to it.

X. PRIZE QUESTION 19.

By Joseph Clay, Philadelphia.

Rittenhouse's hygrometer is formed of two thin pieces of wood, of the same uniform breadth, thickness and length, glued together; the grain of one piece running with the length of the hygrometer, and that of the other with the breadth. The contraction or expansion of the latter piece causes the hygrometer to assume a curvilinear form, and one end of the hygrometer being fixed the degree of moisture is measured on the curve line generated in a plane by the other end. Required the equation, quadrature, and rectification of this curve.

XI. QUESTION 20.

By Robert Patterson, Philadelphia.

For the best satisfactory solution of which, to be adjudged by the Editor, he offers a prize of ten dollars.

In order to find the content of a piece of ground, having a plane level surface, I measured, with a common circumferentor and chain, the bearings and lengths of its several sides, or boundary lines, which I found as follows:

1. N. 45° E. 40 perches.
2. S. 30° W. 25 ditto.
3. S. 5° E. 36 ditto.
4. West 29.6 ditto.
5. N. 20° E. 31 ditto. to the place of beginning.

But upon casting up the difference of latitude and departure, I discovered, what will perhaps always be the case in actual surveys, that some error had been contracted in taking the dimensions. Now it is required to compute the area of this enclosure, on the *most probable supposition* of this error.

RULES TO BE OBSERVED BY CONTRIBUTORS.

1. All communications must be post paid and directed to Robert Adrain, Editor of the Analyst, Reading, Pennsylvania.

2. Those who wish to have new questions inserted must send a true solution along with each question.

3. Any mathematical question sent to the Editor, with or without a solution, and accompanied by a prize of not less than six dollars in value, shall, if judged admissible, be published as a prize question, with the proposer's name; and the prize shall be awarded, by the Editor, to the author of the best satisfactory solution. But if no such solution shall be furnished within five months after publication, the prize may then be appropriated, by the donor or the Editor, to some other suitable question.

TO SUBSCRIBERS.

The publication of the two first numbers of the Mathematical Museum has been from necessity delayed some time in order to procure the necessary characters from the type foundery. The succeeding numbers may be expected more punctually.